

Numerical Simulation for the Bell Directivity Patterns Determination

Andrzej GOŁAŚ, Roman FILIPEK

*AGH University of Science and Technology
Department of Mechanics and Vibroacoustics
Mickiewicza 30, 30-059 Kraków, Poland
e-mail: ghgolas@cyf-kr.edu.pl*

(received December 4, 2008; accepted September 14, 2009)

The analysis of various sound sources is performed on the basis of their directivity patterns. The literature does not contain any information about directivity patterns of bells that are instruments broadly applied to sacral purposes or to create a certain sound space from the aesthetic point of view. The paper presents the methodology of determining the bell directivity patterns by an example of the Russian bell. This example was applied because exact values of geometrical parameters and measuring data of the bell were available.

The model was created by means of FEA (finite element analysis). It included a coupling between the bell and its surrounding acoustic medium. During the modal analysis, the first three natural frequencies of the bell were calculated, and then, using the harmonic analysis, the directivity patterns were determined for the frequencies. Afterwards, the transient response of the system in selected measuring points was determined. The obtained results are important for bell-founders and architects because thanks to the knowledge of directivity patterns, the constructions supporting the bells can be designed in a better way and the sound propagation can be determined more precisely. The presented method of auralisation of the bell sound makes the cooperation between the designer and the receiver fairly convenient.

Keywords: FEA, directivity patterns, musical instruments, idiophones, bell.

1. Introduction

Bells are the musical instruments named idiophones, i.e. the percussion instruments creating sound by free vibrations [1].

One of the oldest bells are Chinese bells [2] that, thanks to their oval shape, generate sounds of two basic component tones as the result of vibrations.

Bells used since the Middle Ages in the Western culture have a tulip shape. Their external surface can be approximated by a hyperbola [3, 4] if the thin walls are assumed. The natural frequencies of a bell made in the casting process are

often additionally tuned by a proper selection of the material of the inner bell surface. The planned natural frequencies have the following ratios, i.e. 1:2:2.4:3:4, that can be achieved quite well as the result of tuning. It is especially important for the bells harmonious in carillon. The first form of vibrations named hum is not a predominant one if it appears in a registered spectrum, and the sound is audible as for the second form of vibrations called prime. The bell sound is complex and features a component as the minor third (2.4), therefore the sound of harmonic bells can be described as at least specific. LEHR [5] designed and made a set of bells in which the major third occurs (the relation 2.5 instead of 2.4).

In recent years the concern for the bell sound did not decrease. The first truly harmonic bells, for which the first seven components are in the harmonic series, were created by Australian BELL in 1999 [6, 7]. For this purpose, the optimization of the cross-section shape through the modal analysis by means of FEA (finite element analysis) was applied so that only the peripheral forms of free vibrations were induced and then they were accordingly tuned. Thereby a completely new sound of bells was obtained, and the bells could be consonant in carillon using complex harmonies. However, a unique tone quality was partially lost.

The distribution of the acoustic field for the predominant resonance frequencies has not been studied so far, and this problem is featured in the paper. The algorithm inducing auralisation of a virtual bell sound is also presented. This process is a standard method during acoustic modelling of rooms but it has not already been used during acoustic analyses of idiophones.

In contrast to the European standards or newly discovered harmonic bells for which the key stage of making an instrument is its precise tuning after casting, the precise tuning of resonance frequencies is not performed in Russian bells. Some

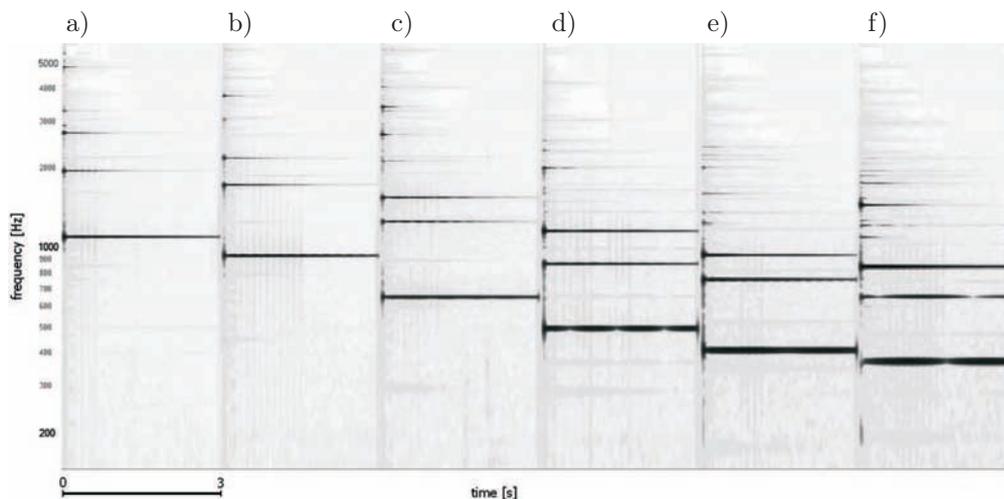


Fig. 1. STFT from sound samples, for bells with mouth diameter and weight: a) 21.5 cm, 6 kg; b) 23.5 cm, 10 kg; c) 27.5 cm, 15 kg; d) 33 cm, 24 kg; e) 40.5 cm, 41 kg; f) 51.5 cm, 92 kg [8].

deviations from the planned resonance frequencies are not cancelled on purpose as they cause disharmonies making the bell sound more intensive and unique. In order to estimate the probable ratios of frequencies, the sound samples together with the catalogue data of one of the bigger Russian bell-foundries were subject to the numerical analysis. Figure 1 presents a short time Fourier transform (STFT) featuring predominant resonance frequencies for the bells cast by Pyatkov & Co., Kamensk-Uralskij in 2004 [8].

Figure 1 proves that the first, the second and the third frequency are predominant for every bell, and their ratios are approximately 1:2:2.4.

2. Test object

The precise data of the bell geometry were kept secret in the bell-founder generations. On the basis of the geometrical ratios between the bell diameter and other dimensions, a bell can be appropriately scaled by tuning its basic frequency up to the planned sound pitch [1]. Data from [8, 9] were applied to this analysis. The bell of the geometry presented in Fig. 2 and of the mouth diameter equal

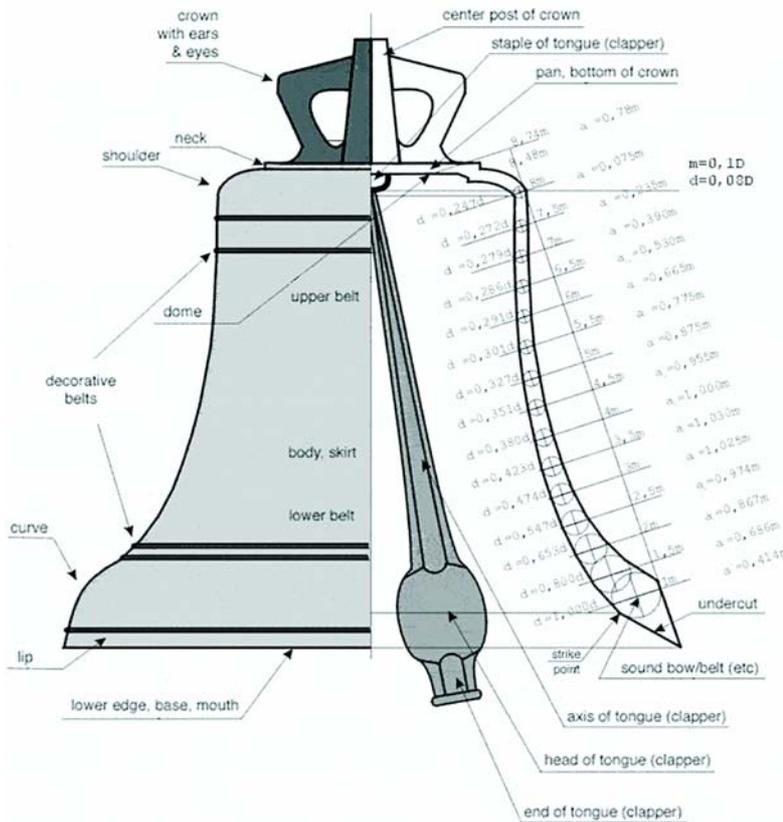


Fig. 2. Basic elements of a bell and geometric parameters (after [8, 9]).

to 1 m were selected as the test object. The material, i.e. bronze, was chosen as isotropic with the following parameters: the Young module of $10.5 \cdot 10^{10}$ Pa, the Poisson ratio of 0.33 and density of $8600 \text{ kg}\cdot\text{m}^{-3}$.

3. Numerical analysis

The bell model (Fig. 3) was created by means of FEA and ANSYS software [13]. The bell surface was modelled using eight-node Shell281 elements. The mesh laid evenly on the surface consisted of about 850 tetrahedral elements of the appropriate thickness in nodes located at the same height.

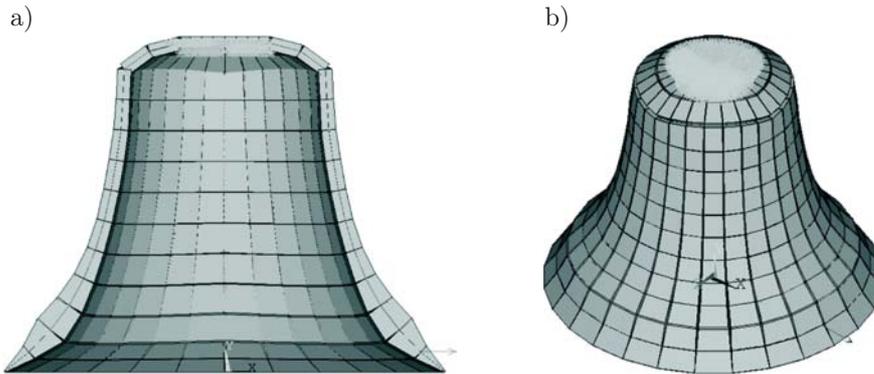


Fig. 3. FE Model with visualized shell thickness: a) the bell cross-section, b) isometric view.

During the first stage of calculations, the natural frequencies and the shapes of own forms within the frequency range from 1 to 1024 Hz were determined. The model was unconstrained, thereby the first six natural frequencies denoting translations or rotations of a solid were rejected during numerical calculations. Through comparison of mode shapes obtained by means of FEA and the mode shapes from the literature [10], the mode shapes affecting the bell sound were determined. The results are presented in Fig. 4 and Table 1.

The differences between the assumed ratios of every frequency and the ratios obtained in the model do not exceed 11%. After expressing the difference between the frequencies in cents, it is evident that in musical terms the interval between hum and prime modes differs from perfect octave by about 2 semitones (201 cents), while interval between minor third and prime differs from perfect minor third by about 3/4 semitones (76 cents). These discrepancies have a strong influence on the sound of the bell, but similar offset of each component of the sound spectrum can be found in the literature [1, 11].

The next stage of analysis was determining sound radiation of the bell. Therefore, the acoustic volume [12] as the sphere of 2 m radius was added to the bell surface. The acoustic medium was air of density of $1.2 \text{ kg}\cdot\text{m}^{-3}$ and sound veloc-

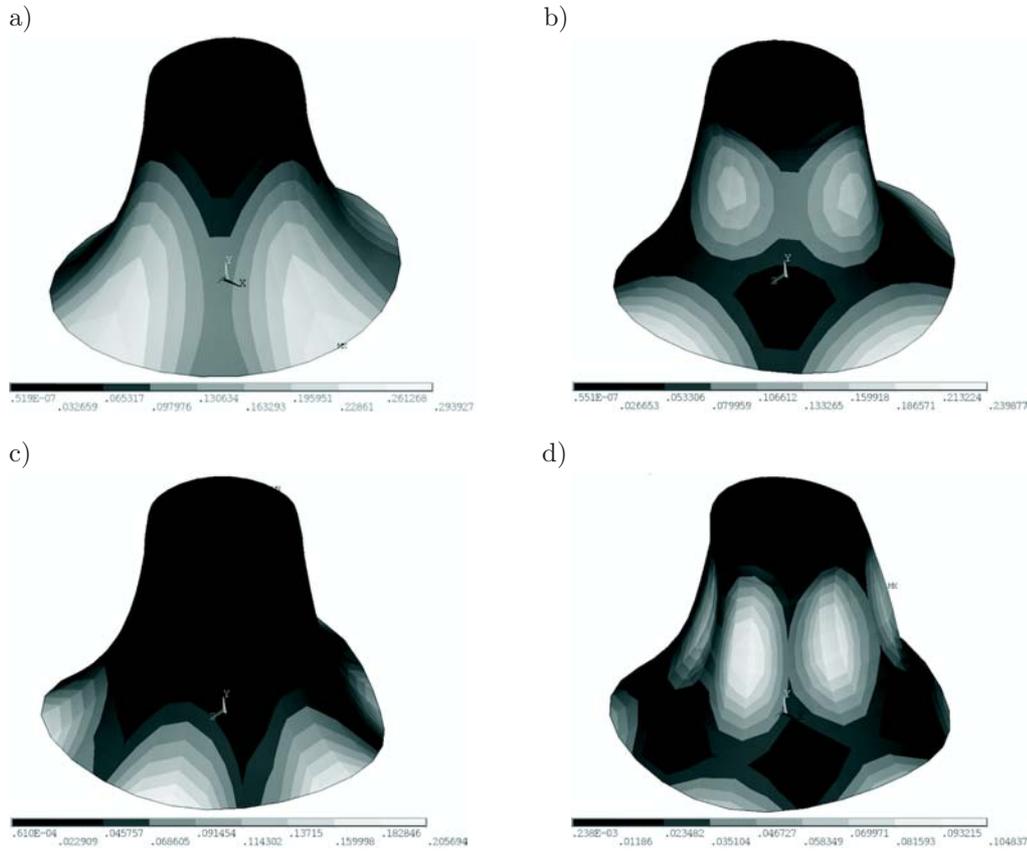


Fig. 4. First four mode shapes of the bell (amplitude of normalized nodal displacements):
 a) hum (2,0), b) prime (2,1#), c) minor third (3,1) and d) fifth (3,1#).

Table 1. First three resonance frequencies of the Russian bell.

| Mode shape / frequency ratio | (2,0) / 0.5 | (2,1#) / 1 | (3,1) / 1.2 |
|----------------------------------|-------------|------------|-------------|
| Name of partial | Hum | Prime | Minor third |
| Perfectly tuned frequency [Hz] | 225.7 | 451.4 | 541.7 |
| Numerical obtained frequency[Hz] | 253.5 | 451.4 | 565.9 |
| Difference [%] | 10.97 | 0 | 4.27 |
| Difference [cents] | 201.1 | 0 | 75.7 |

ity of $343 \text{ m}\cdot\text{s}^{-1}$. The volume was filled with tetrahedral and four-node (of the first order) acoustic elements, named Fluid30 and based on the following wave equation:

$$\frac{1}{c^2} \frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 0, \quad (1)$$

where c – sound velocity, P – acoustic pressure ($P = P(x, y, z, t)$), t – time.

The medium, wherein the wave propagation occurs, meets the assumptions of: compressibility, inviscosity, no average flow, homogeneity of average density and pressure. The Fluid30 elements can be also cubes that guarantee more accurate results for a similar mesh density; however, it is far more difficult (if possible) for them to make a mesh inside the volume of a complex shape.

The Fluid130 elements that conserve the Sommerfeld condition causing the complete sound radiation from the system (with no reflections from the external surface) were situated at the external sphere surface. By means of notation from [13], the Sommerfeld condition can be written as follows:

$$\lim_{r \rightarrow \infty} r \frac{d-1}{2} \left(P_r + \frac{1}{c} \dot{P} \right) = 0, \quad (2)$$

where r – distance from the origin of coordinates, P – acoustic pressure, P_r – pressure derivative along the direction of radiation, $d = 2, 3$ – the number of space dimensions.

The unlimited space was cut by setting an absorbing edge at a certain distance from the structure. The Sommerfeld condition is satisfied, if the elements modeling the edge are located on a circle (as for two-dimensional problems) or on a sphere (as for three-dimensional problems).

The structure and acoustic coupling were considered within a single computation step during calculations. It was possible thanks to the fact that the applied Fluid30 elements being in the contact with the bell surface, had (besides pressure) extra degrees of freedom as displacements. The coupling of displacement field and acoustic field occurs in a matrix equation featuring the element.

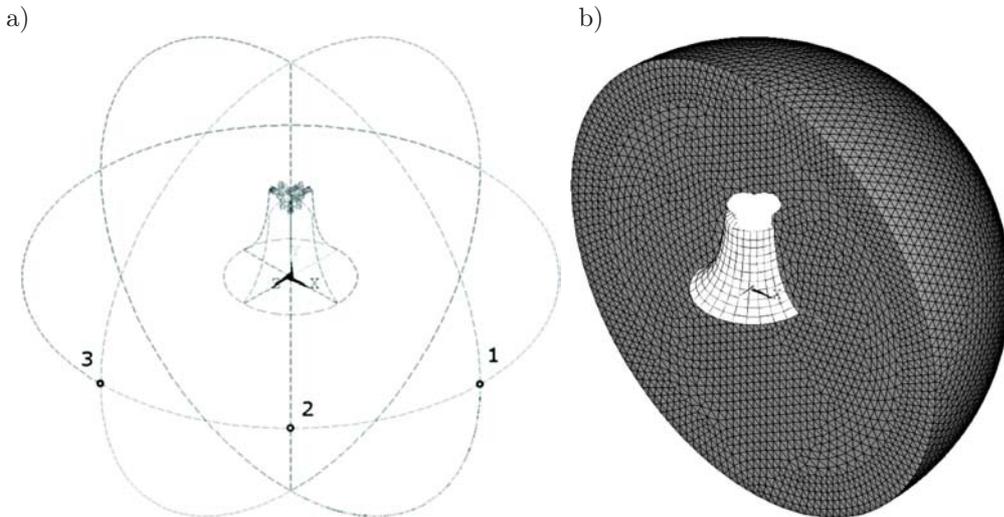


Fig. 5. FE coupled field model: a) Geometry with control points, b) FE Mesh.

The harmonic analysis was performed within the frequency range from 0.5 to 640 Hz with 0.5 Hz spacing. The damping coefficient of $2 \cdot 10^{-4}$ was used in the system. The input function was a harmonically alternating force of 1 N amplitude applied at the node as it was pointed in Fig. 3b. The degrees of freedom related to a displacement and a rotation were fixed at the top surface of the bell. The boundary conditions of the model are similar to a real mechanical system in carillon, wherein the bell is usually immobile and the stroke is generated by a mechanism operating the clapper. The amplitude of the acoustic pressure for the analysed frequency range was calculated in three control points (Fig. 5a) and presented in Fig. 6.

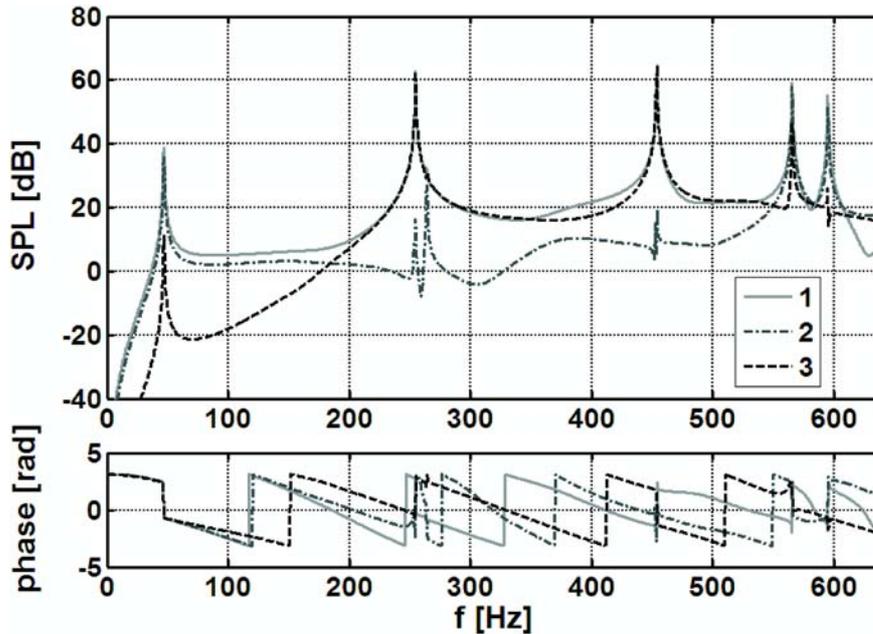


Fig. 6. Sound pressure level and phase angle of acoustic pressure in three control points.

The results of the harmonic analysis for individual resonance frequencies were used to determine the directivity patterns of the system, i.e. distributions of the acoustic field for the resonance frequencies of 254.5, 454 and 565.5 Hz and are presented in Fig. 7–9.

Figure 6 features the resonances occurring not only for the frequency of the input function close to the natural frequency of the bell, but also for the frequency about 47.3 and 595 Hz. They are specific for the assumed boundary condition of the bell's model for which all translational and rotational degrees of freedom are fixed to zero at the top surface of the bell. Then, the extra natural frequencies of the system occur, and in case of a free hanging or hanging of a certain stiffness and attenuation they would not be observed.

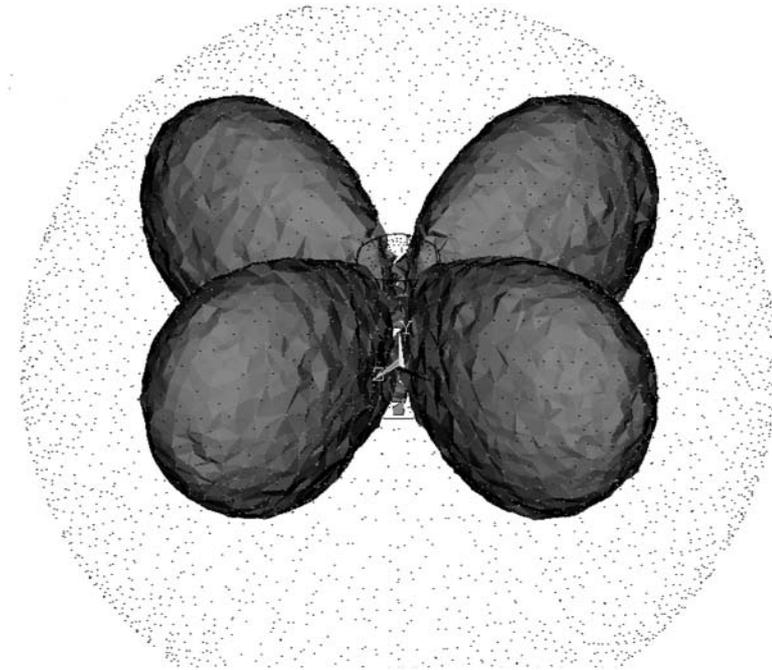


Fig. 7. SPL Isosurface 64 dB for resonance frequency 254.5 Hz.

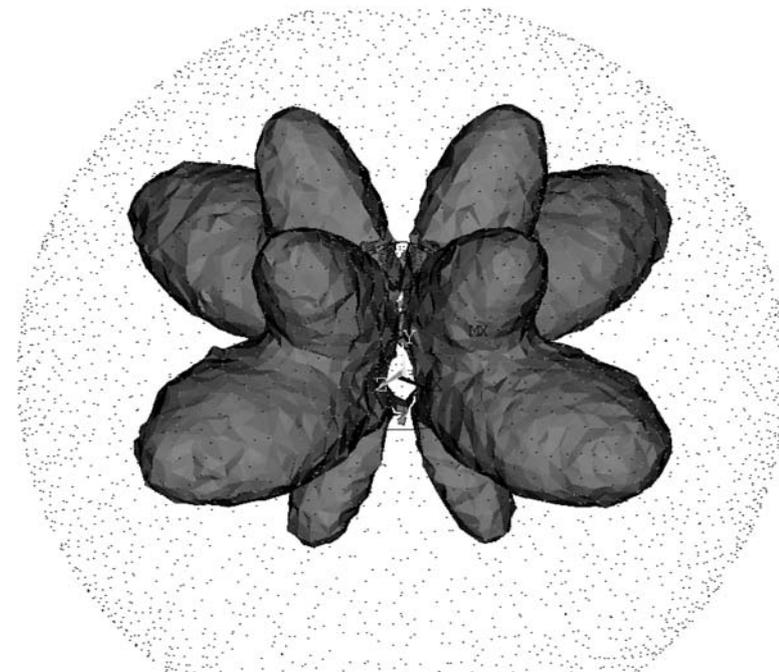


Fig. 8. SPL Isosurface 64 dB for resonance frequency 454 Hz.

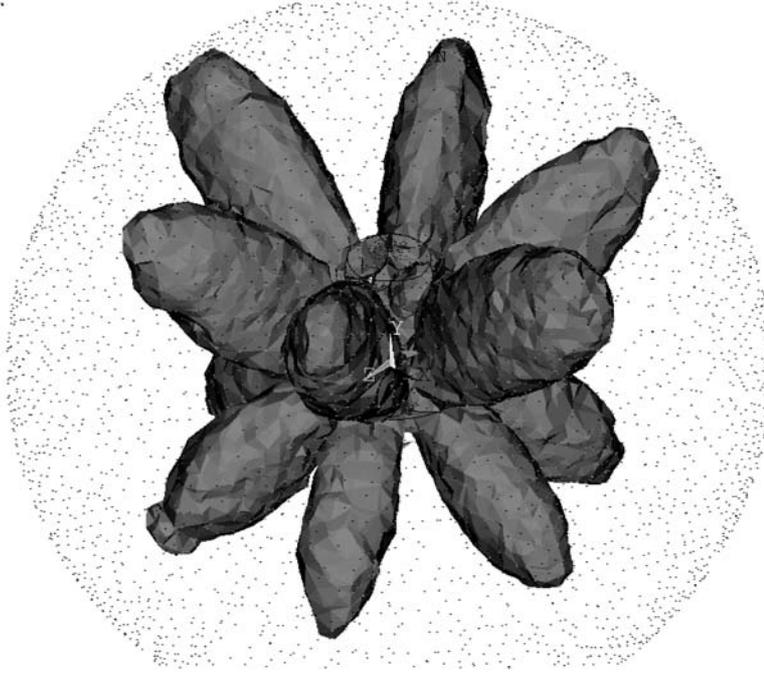


Fig. 9. SPL Isosurface 75 dB for resonance frequency 565.5 Hz.

4. Modeling the transient response of the system

Assuming that the tested system (Fig. 10) is linear and its parameters are time-constant, the term of the impulse response can be used during its analysis.

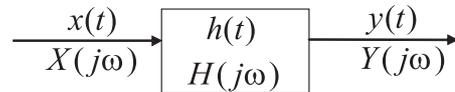


Fig. 10. The assumed linear system, where: $y(t)$ – signal at the system output, $x(t)$ – input signal, $h(t)$ – system impulse response, $Y(j\omega)$, $X(j\omega)$ – Fourier transforms of the input and the output signal, respectively, $H(j\omega)$ – spectral transfer function of the system.

The impulse response of the system explicitly identifies the system. The Fourier transform of the system impulse response guarantees determining of the amplitude and frequency characteristics. If a delta function $\delta(t)$ is given at the system input, the signal at the system output has the form of the impulse response. Having the impulse response $h(t)$, the response of the system $y(t)$ for any input signal $x(t)$ can be obtained as the convolution of input signal $x(t)$ and impulse response $h(t)$:

$$y(t) = x(t) * h(t). \quad (3)$$

According to the properties of the convolution of two functions and the fact that the Fourier transformation is the special case of the Laplace transformation, it can be written as follows:

$$Y(j\omega) = X(j\omega) \cdot H(j\omega), \quad (4)$$

where $Y(j\omega)$, $X(j\omega)$ – Fourier transforms of the input and the output signal, respectively, $H(j\omega)$ – spectral transfer function of the system.

The spectral transfer function of the system was calculated for the numerical model featured in Sec. 3. The result of the analysis, i.e. the acoustic pressure in a given control point, was transferred into Matlab 7.0 in which further calculations were performed. The impulse response of the system was computed using the inverse Fourier transformation. The transient response of the system was determined as convolution of the assumed input function (applied force) and the impulse response. The sample results are presented in Fig. 11.

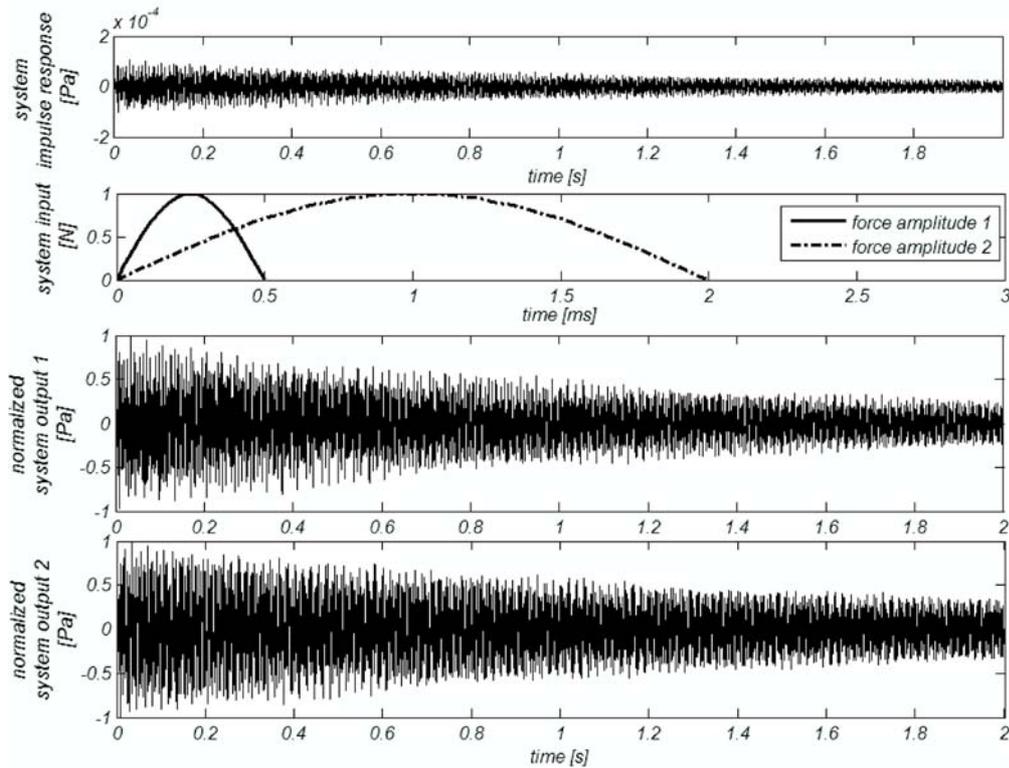


Fig. 11. System impulse response: input signal and output signal for two cases of force amplitude signal with time duration: 1) 0.5 ms, 2) 2 ms.

The characteristics featuring the normalized amplitude of the acoustic pressure as a function of time (Fig. 11) proves that duration of the force impulse has a significant impact on the acoustic response of the system. If the force impulse

is longer, the lower resonance frequencies of the system are stronger induced and this effect is noticeable even in a function of time. The frequencies can be calculated more accurately through the spectral analysis, the Fourier transformation or any filtration in the time domain.

The obtained signals were subjected to standard frequency sampling for reproducing sound records, i.e. 44.1 kHz, and saved as PCM (Pulse Code Modulation – a *.wav file). Thereby, a future reproduction of records by means of any software is possible.

5. Conclusions

The results obtained for the Russian bell can be generalized for a broad group of the third-minor bells because of similarity of the pattern of node lines.

The obtained distribution of the acoustic field for a basic frequency of the bell features a significant irregularity. The sound radiation occurs mainly in a horizontal plane of the bell (in directions perpendicular to the side bell walls) and is the most intense in four directions according to vibration antinodes. The analysis of the first directivity pattern proves that in order to obtain the balanced sound amplification of the space surrounding the bell, a new mechanical construction should be considered. The construction would actuate a clapper (or two clappers) so that alternate strokes in points shifted to each other by 45 degrees in a horizontal plane could occur. Probably, an interesting acoustic effect would be also achieved in the carillon groups of bells if a traditional point of the clapper striking at the gong was changed.

The field distribution is even more non-uniform for a higher resonance frequency. On the basis of the analysis results it can be concluded that any measurements of the real object shall be performed in many measuring points and the strongest acoustic effect is observed in the motion plane of the clapper. The key element of the analysis is determination of the boundary conditions. The velocity of vibrations is specified usually during a stroke. According to the rules of modelling the strike noise, the response of the structure and acoustic system to the input function, i.e. applied force (acceleration) as the signal of determined characteristics within the time domain, was obtained in this model.

The above-mentioned algorithm can be valuable for an architect, as thanks to the knowledge of directivity patterns, the constructions supporting the bells can be designed in a far better way, and the sound propagation path in a space can be determined more accurately. In existing constructions, the knowledge of directivity patterns in combination with sound propagation modeling, for example by geometrical methods [14], can be useful for proper choice of sound recording points.

At the current stage of research, the geometrical and material properties of the bell were chosen from the literature. Discrepancies between the frequencies ob-

tained by numerical simulation and by measurements of the real bell may exceed 20%. The model validation is necessary and will be carried out experimentally. The numerical model with parameters which correspond to the real bell parameters will be computed and then the results will be compared with the real bell measurements.

In further research, the sensitivity of natural frequencies for the material and geometrical parameters will be investigated. And then, the implemented algorithm can be applied to designing of a bell of a new shape and its tuning. The results of calculations were used for auralisation of the bell sound, so reproduction of sound and any corrections can be already performed during making a prototype. Moreover, the sound auralisation can be especially useful for presentation of calculation data for a receiver or a composer who usually does not have any basic engineering or acoustic knowledge, and for whom the graphical interpretation of data could be very difficult.

Finally, virtual reconstruction of sound of the big bells, that got lost or were destroyed, would be also possible. It can be achieved at rough estimate when only basic parameters are known, as in a case of the biggest world bell – the Burmese bell of the king Dhammazedi, cast in 1484 and of 600 tons weight according to the annals. Better approximation of sound can be obtained when the geometrical and material parameters can be measured. For example, the Russian Tsar-Kolokol III bell of 180 tons weight can be considered. It ruptured during the fire in 1737 before it generated any sound and had not been recast but placed on the Moscow Kremlin ground. At the current stage of research, results include large amount of error, but it will be minimized in a further sensitivity study. After identification of the most important bell parameters, the sound of lost bells can be reconstructed more precisely.

Acknowledgment

The numerical calculation were carried out in the Academic Computer Centre CYFRONET AGH within the grant number MNiSW/IBM_BC_HS21/AGH/066/2007.

References

- [1] FLETCHER N.H., ROSSING T.D., *The Physics of Musical Instruments*, Springer 1998.
- [2] ZHANG HUAI Y.Z., YANG CHANGCHUN, SHI YAOLIN, *Study on excitation of the two-tone acoustic characteristic of the chime bell of Marquis Yi of Zeng by finite element method*, Chinese Science Bulletin, 2007.
- [3] RAYLEIGH LORD, *On Bells*, The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science, **5** (1890).

-
- [4] PERRIN R., SWALLOWE G.M., *Rayleigh's Bell Model Revisited*, Stockholm Music Acoustics Conference, Stockholm, Sweden 2003.
 - [5] LEHR A., *The designing of swinging bells and carillon bells in the past and present*, Athanasius Foundation Astens 1987.
 - [6] <http://www.ausbell.com/>.
 - [7] McLACHLAN N.M., HASELL A.G., *Bells*. USPatent, Ed. 2005.
 - [8] www.russianbells.com.
 - [9] KAMINSKI J., *Kolokol: Spectres of the Russian Bell*, PhD. Thesis, University of Technology, Sydney 2006.
 - [10] FLETCHER N.H., *The nonlinear physics of musical instruments*, Rep. Prog. Phys., **62**, 723–764 (1999).
 - [11] SANKIEWICZ M., BUDZYŃSKI G., *Analizy dźwięku największych dzwonów w Polsce*, Otwarte Seminarium z Akustyki OSA'96, Ustroń 1996.
 - [12] FILIPEK R., WICIAK J., *Active and passive structural acoustic control of the smart beam*, The European Physical Journal – Special Topics., **154**, (1), 57–63 (2008).
 - [13] *Theory Reference (Release 11.0 Documentation for ANSYS)*.
 - [14] GOŁAŚ A., SUDER–DĘBSKA K., FILIPEK R., *The influence of sound source directivity on acoustics parameters distribution in Kraków Opera Hall*, Acta Physica Polonica B (in review).