The design of bells with harmonic overtones

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The design of bells with harmonic overtones

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Musical bells have had limited application due to the presence of inharmonic partials in the lower part of their acoustic spectra. A series of bells has been designed that contains up to seven partial frequencies in the harmonic series beginning at the fundamental frequency. This was achieved by choosing geometries for finite-element analysis models in which as many purely circumferential bending modes as possible occurred at frequencies below any mode with an axial ring node. The bell models were then fine tuned using gradient projection method shape optimization and the resulting profiles were cast in silicon bronze. A range of bell geometries and timbres is analyzed using psycho-acoustic models and is discussed in relation to European carillon bells. © *2003 Acoustical Society of America.* [DOI: 10.1121/1.1575748]

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I. INTRODUCTION

The pre-Christian use of the bell, particularly in Europe, appears predominantly to have been apotropaic (to scare off evil spirits). The earliest bells found in the west, from around 1200 BC, are pellet bells. These were worn on clothes and attached to horse harnessing to act as charms to protect the wearer from unnatural harm.[1] However, it was the reverence the early Christians held for the bell and its uses that ensured its continued rise in importance across Europe. Beginning with bell founding in monasteries, small handbells became transformed into stationary bells. The building of the great Gothic cathedrals raised the bells to greater heights from which bells of increasing size could broadcast their sounds to growing Christian communities.[1]

The first reported attempt to tune the partial frequencies of bells relates to Jacob van Eyck (1540–1657), who worked with François and Pieter Hemony on a commission to cast a carillon for the Wijnhuis tower in the town of Zutphen in 1643.[2] The carillon was considered markedly superior to all previous carillons. After the Hemony brothers died, the understanding of tuning of bells fell into disarray and was lost.[2] Bell tuning was recovered from the collaboration between bell enthusiast Canon Arthur B. Simpson, Rector of Fiddleworth, Sussex and the Taylors Bell-founders of Loughborough.[3] Simpson proposed a method of tuning, similar to the Hemony process, where the bell was cast oversize and lathed in particular segments of the waist or the lip of the bell to bring the partial tones into a harmonious series. Their first ''harmonic-tuned bells'' were hung at St. Paul's Church, Bedford, during 1896.[4]

The frequency ratios of the first five radiating partial tones in these Taylors' bells (and common to many tuned European bells since) is 1:2:2.4:3:4. Traditionally these partials are called the ''hum,'' ''fundamental,'' ''tierce,'' ''quint,'' and ''nominal.'' Thus, the European bell could be tuned to a series of partials that included the first four harmonics. However, the presence of the tierce is likely to create complex acoustical percepts of pitch and dissonance that change in time as partial frequencies decay at varying rates. In their discussion of the pitch perception of European bells, Schouten and 't Hart[5] remark, ''If one concentrates on these partials when listening to a ringing bell, it is a remarkable experience to hear them with every strike. The minor third is quite conspicuous and it diminishes only slightly in loudness between strikes.''

The strike tone of the European bell is usually reported as the fundamental and has been shown to be largely influenced by the frequency of the 5, 6, and 7th overtones when subjects attempt to nominate a pitch for the bell.[6] This synthetic percept is an example of a virtual pitch or complex tone sensation arising from a different listening attitude to concentrating on individual partials. It will vary depending on the type of clapper and on how hard the bell is struck.

The pitch of bells can also be described by the salience and multiplicity of simultaneously audible tones and pitch sensations at various times in the bell sound. Mathematical models of pitch multiplicity have been developed and compared to human subjective responses to synthesized bell timbres and a range of musical chords and complex harmonic tones. The calculated and reported values of multiplicity for these synthesized bell sounds were found to be higher than for complex harmonic tones with comparable fundamental tone frequencies.[7] These models first calculate the audibility of pure tones (levels above the masking threshold) and any virtual pitch tones that may then arise. The multiplicity is calculated by first normalizing the audibility of these tones to the most audible tone, summing the normalized tone audibility, and then dividing by the audibility of the most audible tone. A factor determined from results of human subjective tests can then be used to adjust for the listening attitude, where analytical listening favors hearing individual partials. The tone salience of individual pure-tone or complex tone percepts is proportional to their calculated audibility.[7]

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Bells do not naturally produce harmonic overtones because, unlike air columns and strings that vibrate predominantly in one dimension only, bells vibrate flexurally in three dimensions. Flexural vibrations are much more difficult to describe analytically than longitudinal vibrations, and it is common to use numerical methods such as finite-element analysis (FEA) to predict the behavior of bells and gongs.[8,9] A number of major third carillons (where the partial is tuned to 2.5 times the hum tone frequency instead of 2.4) have been designed using finite-element analysis in the Netherlands.[10]

II. OPTIMIZATION METHOD

Given the ability to predict natural frequencies from computer models of bells, gradient projection method shape optimization can be applied to the models to adapt wall profiles and, where possible, arrive at specific tuning ratios of partial frequencies. Predeveloped, commercial FINITE-ELEMENT (FE) software was employed to implement a classic linear finite-element method to numerically determine the natural frequencies of the model.[11] The "solid property" elements used for the modeling consisted of *tetra* and *penta* elements in which the nodes' coordinates determine some subsequent shape parameters such as thickness along the bell length. The accuracy of FEA-predicted frequencies for the lower frequencies of bells has been shown to be around 1% of measured results[9] when working with two-dimensional FE models. It is reasonable to expect greater accuracy from three-dimension solid property models.

A gradient projection method[12] was also utilized in this software in order to optimize the objective parameters by changing the coordinates of the FE nodes. This software is designed to compute the nodal vector sensitivities to the objective parameters (such as modal natural frequencies) as a function of the FE nodes' coordinates. The user may select a zone of active nodes with coordinates that will vary during optimization. The sensitivity is calculated from differences in the objective parameter after displacement of each active finite-element node. The process of optimization then iterates towards a target in accordance with geometric constraints that preserve shape parameters of the model such as symmetry about the vertical axis.

To achieve the final goal of optimization (in this case, tuning the modal frequencies of the first seven modes to the harmonic series), the optimization process would ideally be performed separately on each vibrational mode. Behavioral constraints can be applied to limit the allowable changes of frequency of particular modes so that optimization processes on other modes do not change them. These behavioral constraints are met by computing the nodal vector sensitivities to the constraint parameters as a function of the nodes' coordinates. The final displacement vector for each active FE node is calculated from sensitivities to both constraint and optimization parameters during each iteration of the optimization process. The optimization process stops if progress toward the optimization target cannot be achieved without altering the constrained parameters beyond a given tolerance.

The user sets the step size used in the first iteration of the optimization process. It is also possible to define a reduction rate for the step size in the following iterations in order to prevent the optimization from overshooting the target and then alternating on either side of it. The initial step size and its reduction rate were carefully selected based on experience with similar models.

III. SHAPE OPTIMIZATION OF BELLS

It was quickly observed that the behaviors of many bending modes are strongly correlated with respect to changes in the geometry of the model. Therefore, it is often not possible to raise or lower any given partial frequency using gradient projection method optimization applied to positions of FE nodes without affecting other partial frequencies. Constraining all the modes except the mode being optimized restricts the optimization process, and in most cases the objective value cannot be achieved.

A number of strategies may be employed to overcome this problem. The maximum vibrational amplitudes of the bending modes to be tuned may occur in different locations along the length of the bell, and therefore the magnitude of their sensitivity vectors may also vary. It may be possible for the constrained modes to remain relatively unaffected by the optimization process if a careful selection of the active nodal zones is made on the basis of the relative positions of vibrational maxima.

A study of the behavior of modes for changes in model geometry could enable the user to identify which mode types are correlated for certain geometrical changes. By careful selection of the sequence of modes to be optimized, and the constraint sets, groups of highly correlated modes could be moved simultaneously toward a set of target frequencies. Furthermore, the deviation tolerance of the constrained modes' frequencies may be increased to allow them to change more during the optimization. This may be sufficient

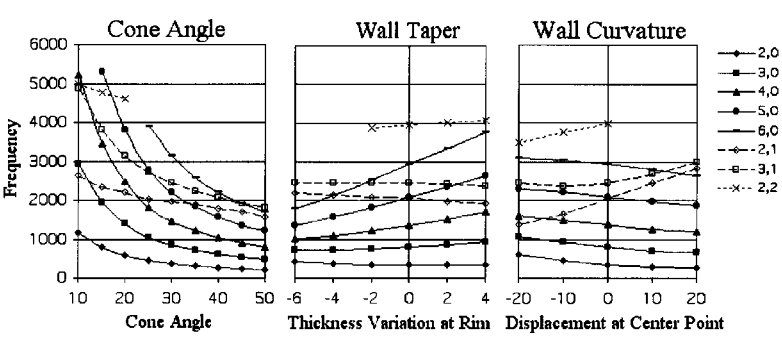

FIG. 1. FEA-predicted frequencies for varying the cone angle, wall taper, and wall shape of freely vibrating capped cones.

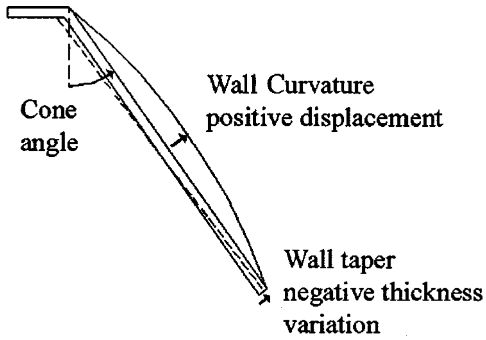

FIG. 2. The geometric parameters used to produce the data shown in Fig. 1.

to achieve a set of optimization targets within an acceptable error.

In order to better understand these correlated behaviors a series of experiments was conducted in which only one geometric parameter of an FE model was varied, and the resultant changes to the predicted partial frequencies of various modes were tabulated and plotted. The effects of geometric parameters such as wall curvature and taper, and cone angle for cylinders and cones with closed ends were investigated for models able to vibrate freely. The results for varying cone angles have been reported in a previous paper.[13] Results for purely circumferential modes (modes which produce only nodal lines in plane with the axis of symmetry of the bell), and mixed modes (producing both nodal lines and rings) are reported below.

Wave number pairs (m,n) are used to describe bending modes such that the first number refers to the number of nodal lines in plane with the axis of symmetry (i.e., circumferential wave number), and the second number refers to the number of nodal rings perpendicular to the cylinder length (i.e., axial wave number). The data from the experiments referred to above could then be used to select a model geometry in which the partial frequencies were reasonably close to the harmonic series and, given an understanding of the behavior of the modes, likely to be able to be tuned correctly using gradient projection method shape optimization. Figure 1 shows plots of FE predicted frequencies for varying cone angle, wall taper, and wall curvature of a capped cone of 200-mm length, 220-mm top circumference, and 10-mm wall thickness. These parameters are defined in Fig. 2. Silicon bronze (95% copper, 1% manganese, and 4% silicon) and its material properties were used for all the models and cast bells described in this paper. The mass density and Young's modulus of silicon bronze are 8.4×10^3 kg/m^3, and 9.4×10^{10} N/m^2, respectively.

From this figure it can be seen that the frequencies of the m,0 modes remain approximately equally spaced as the range of their frequencies change with these geometric changes. At a cone angle of 10° the modes are in the order 2,0; 2,1; 3,0; 3,1; and 4,0, which is the order found in European bells. Mixed modes decrease in frequency more slowly than purely circumferential modes with increasing cone angles and increasing wall tapers toward thinner rims. Increasing curvature away from the axis of symmetry (concavity) increases the frequency of the mixed modes and decreases the frequency of purely circumferential modes.

Figure 1 reveals that the modes with the same number of axial nodes (n) are generally correlated in their frequency responses to changing geometry. This is most evident in the behavior of the m,0 modes described above, but is also clear from the data for the m,1 modes shown in Fig. 1. In setting a frequency optimization target for any given vibrational mode, it was clear that the correlated modes would behave in a similar fashion. For example, it was not possible to substantially change the frequency of the 3,0 mode while constraining the 2,0 and 4,0 modes. As can be seen from Fig. 1 the presence of an m,1 mode between two m,0 modes creates a very uneven distribution of frequencies. Given these modal behaviors, it was realized that to tune the partials to the harmonic series using gradient projection method optimization it would be necessary to first separate the mixed modes from the m,0 modes. The simplest geometrical solution to achieve this was high cone angles and a tapered wall thickness to a thinner rim.

Table I shows the results of optimization starting from a simple truncated cone with a tapered wall and resulting in a harmonic bell with the profile shown in Fig. 3. The entries shown in column 1 of Table I refer to the three geometric models involved in the optimization process, and column 2 refers to the optimization settings required to arrive at that geometry. For example, *down1 constrain3* means drop the frequency of mode 1 in geometry 1 to the target frequency shown for geometry 2, while constraining the frequency of mode 3. From the results of the first stage of the optimization, it can be observed that expanding the frequency range between the 2,0 and 4,0 modes (by dropping the frequency of mode 1 while constraining mode 3) causes a similar expansion in the frequency range between the 4,0 and 7,0 modes. Since the 4,0 mode was constrained, this raised the

TABLE I. Sequential optimization results using "RESHAPE" for the tuning of a 7-partial, harmonic bell starting from a truncated cone with tapering wall thickness.

Step	Mode no.	1	2		3		4		5		6		7	
	Mode type	2,0	3,0		4,0		5,0		6,0		7,0		2,1	
		Freq	Freq	Ratio	Freq	Ratio	Freq	Ratio	Freq	Ratio	Freq	Ratio	Freq	Ratio
1	Initial tapered cone	188	370	1.97	540	2.87	704	3.74	871	4.63	1060	5.64	1260	6.7
	% error			−1.6		−4.3		−6.4		−7.3		−6		−4.3
2	Down 1 constrain 3	180	357	1.98	538	2.99	727	4.04	925	5.14	1141	6.34	1256	6.98
	% error			−0.8		−0.4		0.97		2.78		5.65		−0.3
3	Down 6 constrain 1,2,3,7	181	359	1.98	535	2.96	711	3.93	887	4.9	1079	5.96	1255	6.93
	% error			−0.8		−1.5		−1.8		−2		−0.6		−0.9

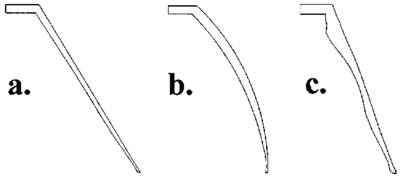

FIG. 3. (a) FE profile of a 7-partial, conical, harmonic bell; (b) 7-partial, concave, harmonic bell; and (c) FE profile of a 5-partial, conical, harmonic bell.

frequency of the higher frequency m,0 modes. The 2,1 mode, however, was only slightly affected. A slight correction of the higher m,0 mode frequencies was then possible in the final stage of the optimization.

A maximum allowable tuning error for the overtones was set at 2% from the harmonic series. Human frequency discrimination of pure tones of 0.5-s length and between 125 and 1000 Hz varies between 0.2% and 1.7% depending on the frequency and the method used to determine the just-noticeable difference.[14,15] For pure tones 10 dB above a partially masking broadband noise, the just-noticeable frequency difference doubles and continues to increase for louder masking noise.[16] Since all the partials heard in a bell are partially masking each other, a tuning error of less than 2% for the upper partials was considered reasonable. The probability that a mistuned harmonic is detected as being different from the harmonic series by automatic neural processes has been shown to increase substantially for mistunings greater than around 4% as determined by event related potentials (ERP) in EEG recordings. Subjective responses and ERP attributed to active decision-making processes for hearing the mistuned harmonic as a separate auditory entity increase substantially from a probability of around 10% for mistunings of the second harmonic greater than 2%. The probability of segregating the mistuned harmonic generally decreased for higher harmonics.[17]

IV. DESIGN VARIANTS

The profile shown in Fig. 3(a) (profile 1) will produce a fundamental frequency of 220 Hz for a bell with a diameter of 395 mm, height of 248 mm, and mass of 19 kg. European tuned bells with the same fundamental frequency (hum tone) and a strike tone of 440 Hz are about twice the diameter and weigh about 250 kg or more depending on the profile. As a consequence, European bells of the same fundamental frequency are able to produce much greater loudness than bells of this design. To increase the applicability of harmonic bells to a range of musical contexts, design variants were developed that were physically larger for the same fundamental tone.

Figure 1 reveals that creating models with increasing concavity (shown as positive curvature displacement in Fig. 2) increases the frequency of the mixed modes and decreases the frequency of purely circumferential modes. This enabled concave harmonic bells with seven tuned partials to be tuned by this method of shape optimization. These bells have greater mass and surface area than the equivalent conical bell and have been cast with a fundamental frequency of 75 Hz and a mass of 1150 kg. At a fundamental frequency of 220 Hz they will weigh 44 kg. Close inspection of Fig. 3(b) (profile 2) will reveal a small cylindrical section that was added to the bottom of the bell profile to further raise the frequency of the mixed modes relative to the circumferential modes prior to beginning the shape optimization.

The rims of the bells described above are thin relative to the rest of the bell. If these bells were scaled down in size to produce a fundamental frequency of greater than 440 Hz, the rims would be so thin that the bells could be susceptible to damage when struck (frequency is inversely proportional to size if scaled in all three dimensions). Finite-element studies indicated that increasing wall thickness increases the frequency of all bending modes. The frequencies of mixed modes do not increase as quickly as purely circumferential modes in capped cones, and so fewer purely circumferential modes have frequencies below the frequency of the first mixed mode and can be tuned by this method of shape optimization to the harmonic series.

A thicker conical bell was designed and cast with six partial frequencies in the harmonic series to compare with the conical bell with seven partial frequencies described above. This bell had a fundamental frequency of 215 Hz but was 550 mm in diameter, 305 mm in height and weighed 29 kg. Apart from the increased wall thickness, the bell had a very similar profile to the conical bell shown in Fig. 3(a) and is described as profile 1a in Table II.

Since the partial frequencies being tuned are predominantly due to circumferential bending modes, reducing the circumference of the bell at the rim by decreasing the cone angle further increases the frequency of the fundamental without decreasing the length and thickness of the wall (see Figs. 1 and 2). However, the frequencies of the mixed modes do not increase as quickly as the purely circumferential modes, and so fewer partials can then be tuned to the harmonic series by gradient projection method optimization applied to FE nodal positions. Maximizing the number of par-

TABLE II. Frequencies measured by peak detection of FFT spectra.

		Measured frequencies of bells described in figures						
Profile	Mass (kg) @ 220 Hz	Freq 1 (Hz)	Freq 2 (Hz)	Freq 3 (Hz)	Freq 4 (Hz)	Freq 5 (Hz)	Freq 6 (Hz)	Freq 7 (Hz)
1.	19	207	419	613	814	1018	1249	1434
1a.	29	345	692	1033	1362	1709	2062	NA
2.	46	188	380	567	746	924	1116	1313
3.	83	232	465	699	944	1176	NA	NA

tials tuned to the harmonic series resulted in much greater wall tapers with increased wall thickness near the cap of the bell.

Conical harmonic bells with 5, 4, and 3 partial frequencies tuned to the harmonic series have been designed and cast to enable bells with fundamental frequencies as high as 1720 Hz (the note E5) to be produced. These bells have cone angles as low as 13.5 degrees. Figure 3(c) shows the profile of a 5-partial harmonic bell (profile 3). This profile produces a fundamental of 232 Hz for a bell of 520-mm diameter, 460-mm height, and weighing 83 kg. Careful inspection of the profile will reveal that the outside wall is not a straight line as in other conical bells, as the FE nodes comprising this wall were also allowed to move in the optimization process to maximize the tuning possibilities. This profile is expected to be appropriate for church and carillon towers (except for full-circle ringing) as well as for higher frequency orchestral bells.

V. ANALYSIS OF CAST BELLS

Tuning errors were encountered during the manufacture of many bells. These were predominantly ascribed to geometric inaccuracies or variation in the metal properties created in the casting of the bell. It was found to be possible to fine-tune these bells on a lathe after casting, by using shape optimization experiments as a guide as to where to remove material. The frequency of the lower frequency modes could be decreased by reducing the wall thickness near the cap, and increased by reducing wall thickness near the rim. Lower frequency modes could be decreased without affecting higher frequency modes; however, increasing them by thinning the wall near the rim caused the frequency of the higher frequency modes to decrease. All modes increase in frequency if the bell is shortened in length. If retuning was found to be necessary it sometimes resulted in the highest frequency tuned mode falling outside the 2% error margin.

The acoustic spectra of cast harmonic bells with profiles 1a to 3 are shown in Fig. 4. The actual frequencies shown in the spectra of the bells are given in Table II along with the frequencies measured for a bell cast with profile 1. All the tuned frequencies for bells with profiles 1 to 3 are within 2% of the harmonic series. Table II also includes the mass for a bell of each profile when scaled to produce a fundamental frequency of 220 Hz.

The acoustic spectra were produced from recordings taken at 1 m from the bell surface perpendicular to the axis of symmetry of the bells. Bell 1 was struck near the rim by a steel hammer with a hard nylon head of 0.5-kg mass, bell 2 by a steel hammer with a hard nylon head of about 1.5-kg mass, and bell 3 by a spherical cast-iron clapper of about 5-kg mass. These mallets were chosen as appropriate for producing a tone balance predominantly consisting of the full range of tuned partials. The mallet velocity was such as to produce an ''A-weighted'' sound-pressure level of around 85 dB (fast response) in the early part of the sound. The FFT was produced at about 50 ms after the sound onset by using a Hamming window of 4096 samples for a sample rate of 44.1 kHz.

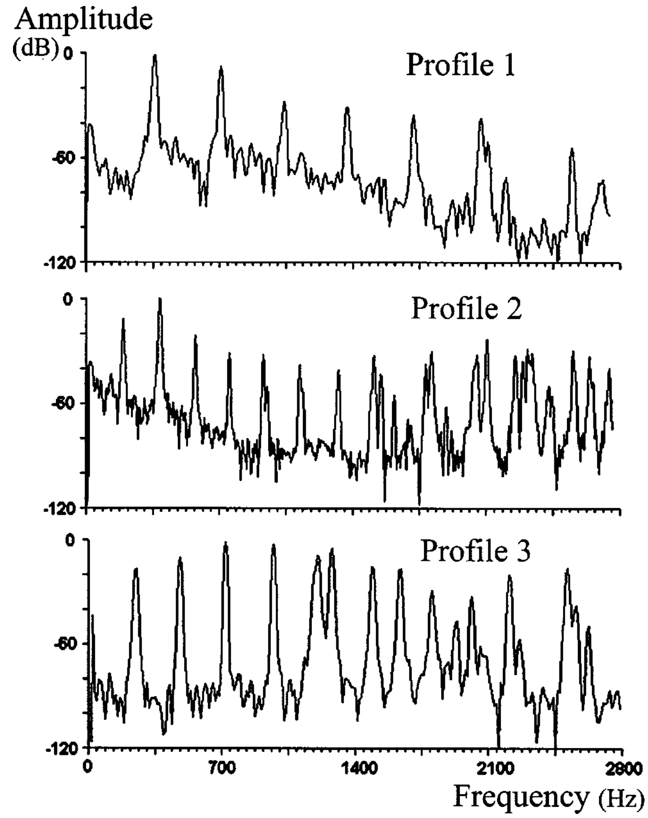

FIG. 4. Acoustic spectra recorded 50-ms. after the sound onset for bells described in Table II.

The strategy of increasing the loudness of conical bells by increasing their mass at the expense of the number of tuned partials has been evaluated as follows. The difference in the perceived timbre of conical bells with 6 or 7 partials was only slight, but the larger and thicker bell was able to produce about 10 dB(A) greater peak sound-pressure level (fast response) for a range of strike velocities between 1 and 2.5 m/s with a typical tubular bell mallet (leather-clad hardwood) or a steel hammer with a hard nylon head and mass of about 0.5 kg. Critical bandwidths increase in proportion to frequency for frequencies above 500 Hz.[18] Therefore, for higher harmonic numbers, and higher fundamental frequencies, the more partial frequencies occur in the same critical band and the greater partial masking will occur between them. Comparing the probability of reporting mistuned harmonics as separate auditory entities from otherwise harmonic complex sounds with fundamental frequencies of 200 and 400 Hz has shown the diminishing importance of higher partials to the perception of pitch. The probability of segregating the sixth harmonic mistuned upwards by 5.3% was reduced from about 80% to about 30% when the fundamental frequency was doubled.[17]

Psycho-acoustic models have been employed to evaluate the bells prior to undertaking more extensive subjective tests relevant to various musical contexts. If the high-frequency untuned partials do contribute more than a general sharpness percept in the early portion of the sound, they will increase the pitch multiplicity of the bell. This was measured using Parncutt's algorithm[7] applied to FFT spectra of the bells calculated using a Hamming window of 2048 samples at a

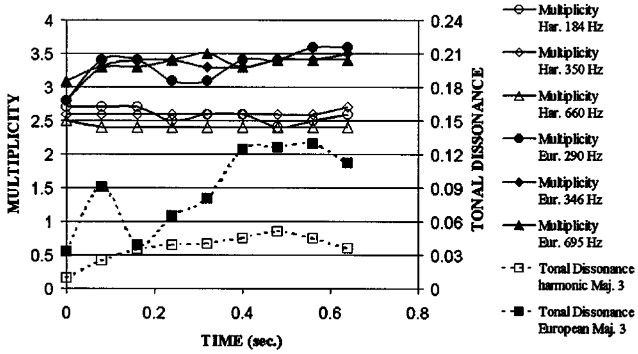

FIG. 5. Comparison of calculated pitch multiplicity and tonal dissonance for a range of harmonic bells and European carillon bells described by the frequencies of their first partial. The harmonic bells at 188 and 345 Hz are described in Table II and Fig. 4 with profiles 2 and 1a, respectively.

sample rate of 12.8 kHz.[19] Pitch multiplicity can be as high as 3 for harmonic complex tones due to the audibility of individual harmonics and the presence of multiple virtual pitches or subharmonic pitch percepts. Figure 5 shows pitch multiplicity calculated over 80-ms intervals from the sound onset for a range of harmonic bells including the bells with profiles 1a and 2 described in Table II, and three European carillon bells with comparable frequencies. The harmonic bell with a fundamental at 660 Hz is a conical 5-partial bell with a profile similar to profile 3. The pitch multiplicity of all three harmonic bells remains around 2.5 for the first 640-ms of the sound, and generally decreases as pitch increases, despite the number of tuned partials decreasing from 7 to 5. The calculated multiplicity over the same time period for the larger 5-partial bell described in Table II is shown in Fig. 6,

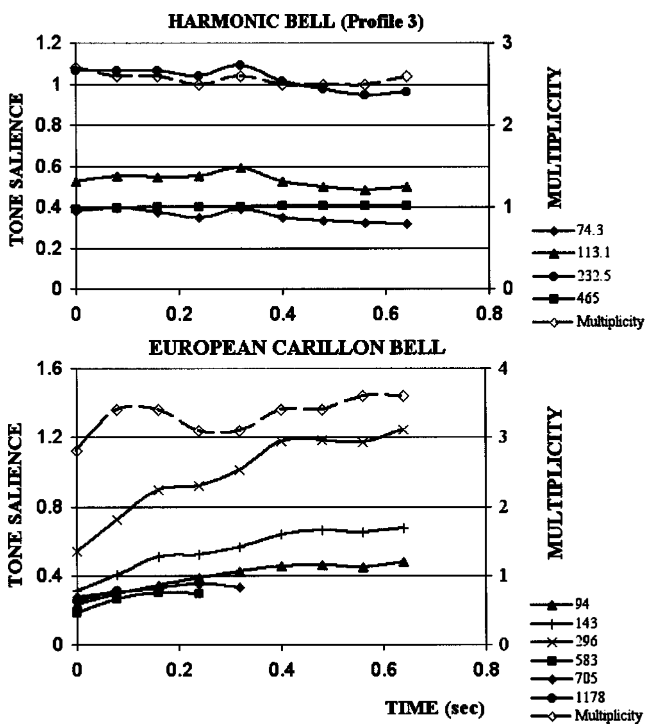

FIG. 6. Comparison of calculated tone salience and pitch multiplicity for a lower pitched 5-tone harmonic bell (profile 3 described in Table II) with a European carillon bell.

and is similar to the harmonic bells described in Fig. 5. This suggests that there is little improvement in the pitch clarity gained by tuning partials beyond the fifth partial for harmonic bells of this pitch.

All the harmonic bells have much lower calculated pitch multiplicity than the European bells (recorded in a similar manner *in situ* at the War Memorial Carillon, Loughborough, UK). The European bells' calculated pitch multiplicity (shown in Fig. 5) increases after the first 80-ms interval, as masking effects in the sharp attack portion of the sound diminish and more partial frequencies become audible. Figure 6 shows the calculated pitch multiplicity and tone saliences over the same time period for the lowest pitch European bell. Tone salience was calculated using Terhart's algorithm.[20] The audibility of the tierce at 705 Hz, the fundamental at 583 Hz, and the nominal at 1178 Hz can be seen to contribute to the calculated pitch multiplicity for the first 300 ms. Thereafter, the hum at 296 Hz and its subharmonics become the principal contributors to the pitch multiplicity.

These results suggest that the perceptual prominence of a single strike tone in European bells lasts for less than 80 ms when the bell is struck with moderate force in the manner of a carillon playing. While the attack portion of musical sounds influences judgments about the rest of the sound, it is likely that large changes in the perceived timbre occurring after the attack portion, such as those shown above for European bells, will also be salient. Figure 6 also shows comparative tone salience and multiplicity results for the heavy 5-partial, harmonic bell described in Table II (profile 3). The contributions to the calculated multiplicity of the second partial at 465 Hz, and the first partial at 232 Hz and its subharmonics remain consistent throughout the first 640 ms of the sound.

Figure 5 also shows the tonal dissonance for major third dyads of harmonic bells and European carillon bells calculated over 80-ms intervals using the algorithm developed by Hutchinson and Knopoff[21] applied to FFT produced as described above. The 290- and 346-Hz European bell sounds were used with the 346-Hz sound pitch shifted up one semitone using a digital sampler. Two harmonic bells of the same fundamental frequencies were also used. Tonal dissonance is calculated from the sum of dissonances between partial tones. This is derived by multiplying the measured amplitudes and frequency differences between partials by roughness factors associated with critical bandwidths determined for each frequency difference. The sum is then normalized by dividing by the total power of these partials. The dissonance of major third dyads calculated for the European bells is consistently greater than for the harmonic bells and varies in a similar fashion to the changing multiplicity of the bell sounds used in the calculation.

Another factor influencing the loudness and perceptibility of high-frequency partials is the internal damping of the bronze. When struck by a hard wooden beater, silicon bronze bells with fundamental frequencies above 1200 Hz produce a fundamental frequency with a sound-pressure level more than 50 dB greater than any of the higher partials when measured at 1-m distance perpendicular to the axis of symmetry. The fundamental will dominate in the perception of pitch in these bells, and possibly mask all other spectral components of the sound. It is therefore appropriate to design bells with as few as three tuned partials as long as they are high pitched. The material damping in tin–bronze bells (for alloys of 10%–22% tin) is less than silicon bronze bells, and so they sustain the resonance of higher partials for longer times. However, tin–bronze is also more susceptible to cracking, especially with the thin rims necessary to produce harmonic bells, and so only one prototype harmonic bell has been cast in tin–bronze to date. European-style handbells are made in tin–bronze and generally only have two tuned partials, although many more are distinctly audible.[22]

VI. CONCLUSION

The combination of gradient projection method shape optimization of FE nodal positions with the FE frequency modal analysis data for variations of a series of geometric parameters enabled bells with harmonically tuned partials to be designed. The design process involved the careful choice of a starting geometry for shape optimization based on knowledge of the frequencies of modes and the correlated behavior of various lower frequency mode types for cones and cylinders with geometries in the range of interest. This process often required the designer to make changes to the initial geometry of the model based on the broadly determined parametric data after a gradient projection method optimization failed to reach optimization targets.

Harmonic bells have been designed and manufactured with less than 2% error for up to the first seven partial frequencies. These bells are expected to expand the musical applications of bells due to reduced complexity of pitch percepts in the sound of the bells and partials likely to create dissonance in musical chords. The effects of the tuning errors of the upper partials will be on the clarity of the perceived pitch when the bell is heard in isolation, and the roughness due to beating partials when played in musical chords. In the bells manufactured to date[23] these affects are very small, but will require more research to properly quantify.

Harmonic bells have been used in a number of musical contexts. Harmonic bells ranging in mass from over 1 ton to less than 1 kg are included in a permanent outdoor installation of electro-mechanically struck bells in Melbourne, Australia. These bells are struck by a variety of mallet materials ranging from steel pins for the smaller bells to metal mallets with hard nylon striking surfaces for the larger bells. They are controlled by MIDI programs and can be played with a range of mallet velocities. This installation is not intended to be as loud as a chime of European bells, but nevertheless can be readily heard from over 100-m distance. Over 2000 harmonic handbells and a two-octave set of larger harmonic bells for the Melbourne Symphony Orchestra were produced for performances to celebrate the centenary of Australia's federation in 2001.[23]

ACKNOWLEDGMENTS

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